CRITERIA AND FORMULAS FOR THE CALCULATION OF THE FLUIDIZATION RATE

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An equation of motion is derived for individual particles in a fluidized bed of a monodisperse material in connection with the onset of fluidization. Dimensionless complexes have been derived on the basis of this equation and these are used to process the experimental data. A theoretical formula is derived.

The great diversity in empirical relationships for the calculation of the critical rate of fluidization [1] is explained by the absence of a conclusively formulated system of equations for the hydrodynamics of the fluidized bed.

Most of the researchers [2–6] directly or indirectly accept the internal problem of hydrodynamics and treat the fluidized bed as a system of continuously varying channels formed by particles of some material with an equivalent diameter d_{eq}. The equivalent channel diameter (d_{ch}) is determined from the equivalent grain diameter. This transition transforms the problem: it ceases to be strictly an internal problem (the grain diameter appears), but it does not consequently become an external problem, since all of the initial premises of the internal problem are retained. Moreover, these premises lead to a situation in which we take as the characteristic complexes $Re \equiv wd_{eq}/\nu$ and $Ar \equiv d_{eq}^3 g/\nu^2(\Delta\rho/\rho)$, derived from the equation of motion for a homogeneous medium. We note that the Ar criterion cannot be derived from the internal problem.

The fluidized bed may be regarded as an aggregate of particles flushed by a stream of gases [5–8, 12]. Here the particle diameter d_{eq} serves as the characteristic dimension of the system. In our opinion, this approach is closest to the physical phenomenon, since the particle diameter is a stable quantity [8].

In examining the "external" problem, we find that equilibrium for the material particles in relative motion sets in when the sum of all forces—including the force of inertia—is equal to zero:

$$\mathbf{P} + \mathbf{G} + \mathbf{F}_f + \mathbf{F}_d + \mathbf{F}_c + \mathbf{I} = 0.$$

Here the forces are per unit of particle volume in the moving system of coordinates of the particle.

We know from the study of the motion of the solid phase [9] that the state of the particles in which they are suspended in the gas stream without motion corresponds to the onset of fluidization. This state for the particles has been assumed in our further discussions.

In an ideal fluidized bed of monodisperse particles of spherical shape, the latter begin to move simultaneously. In this state their absolute velocity of motion u is equal to zero. All of this makes it possible to neglect the term expressing the particle collision force $\mathbf{F}_c$. At the instant of fluidization onset, with critical porosity resulting from the absence of particle collision, the nature of the fluid boundary layer about each particle is defined exclusively by the properties of the fluid and the state of the material surface. There is no turbulization or destruction of the boundary layer resulting from the effect of the adjacent particles (i.e., their collisions). Therefore in the lift term of the equation of motion we can take μ as the viscosity of the fluidizing agent.

The lift force of the particle is composed of the forces of friction and drag whose fraction for an individual particle in the fluidized bed cannot be determined. For an ideal single layer at the instant of fluidization onset the particles are strung out vertically in chains, with a distance between centers of the order of d_{eq}, i.e., with the exception of the very lowest, they are all in the hydraulic shadow [7]. For this reason, in first approximation, we can omit the frontal drag term $\mathbf{F}_d$. The equation for the forces acting on a massy point (an individual particle) in the case of its interaction with an incompressible medium can be written in the form

$$-\operatorname{grad} P + (\rho_s - \rho)\,\mathbf{g} + \mu\nabla^2 v + \rho_s(\mathbf{v}\operatorname{grad})\,\mathbf{v} = 0.$$

We note that the first term does not coincide strictly with the true value of grad P within that volume of the medium containing no particles.

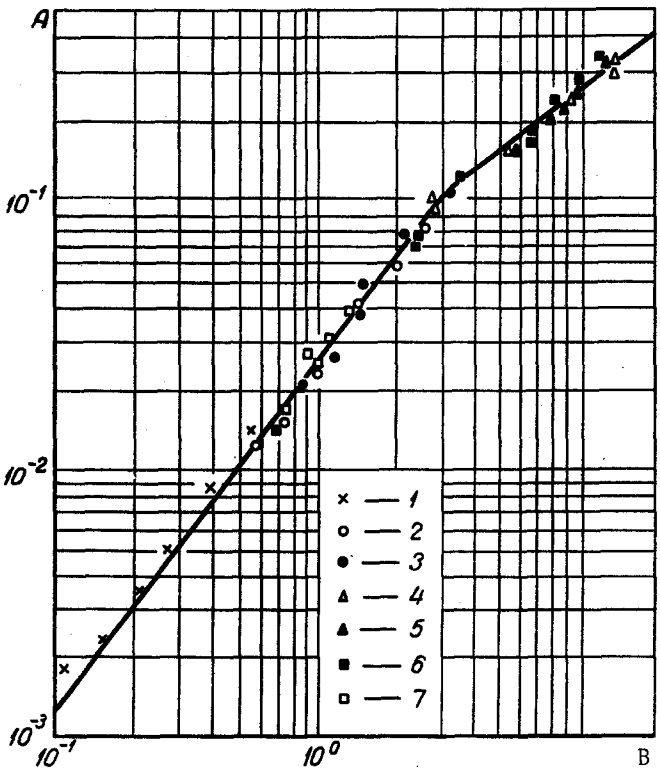
Since we have four terms in the equation for the active forces, we obtain three independent dimensionless complexes

$$\frac{\mathbf{P}}{\mathbf{I}} + \frac{\mathbf{G}}{\mathbf{I}} + \frac{\mathbf{F}_f}{\mathbf{I}} = 1.$$

The complexes obtained after the transformations are reminiscent of the Froude and Reynolds numbers derived from the equation of motion for a homogeneous medium, but are fundamentally different from these:

$$\pi_{IG} \equiv \frac{v^2}{gl}\,\frac{\rho_s}{\Delta\rho}; \qquad \pi_{IF} \equiv \frac{vl}{\nu}\,\frac{\rho_s}{\rho}.$$

In the general case the hydrodynamics of the problem reduces to the determination of the number Π_{IP} as a function of Π_{IG} and Π_{IF}. With regard to the hydrodynamics of the fluidized bed, when we have to determine the critical fluidization rate, the problem becomes more complex since the rate is included in all of the dimensionless complexes but is not specified in the boundary conditions. Let us group the dimensionless complexes so as to eliminate the rate. As a result of the transformations we obtain two dimensionless complexes, one of which is a number while the other is a



Generalization of experimental data [8, 13] on critical rate of fluidization of monodisperse beds at various pressures and temperatures: 1, 2, 3) Fluidization of MCH spheres by hydrogen, air, and carbon dioxide, respectively [8]; 4 and 5) fluidization of silicagel and steel spheres by air [8]; 6) fluidization of alumosilicate catalyst by a gas at a pressure from 1 to 16 atm [13]; 7) fluidization of naturally alloyed iron ore by air at a temperature from 20 to 400° C [8].

criterion. The number is given as the product

$$\pi_{IG}\pi_{IF} \equiv \frac{v^3}{g\nu}\,\frac{\rho_s^2}{\rho\Delta\rho}, \qquad (1)$$

while the criterion is given as the quotient from the division

$$\frac{\pi_{IF}^2}{\pi_{IG}} \equiv \frac{gl^3}{\nu^2}\,\frac{\rho_s\Delta\rho}{\rho^2}. \qquad (2)$$

This method is used in the theory of heat transfer in free motion and it was first applied to heterogeneous systems in slightly altered form by Lyashchenko [11] and corrected in accordance with the transformations of the method from the theory of similarity [10].

The derived complexes contain homogeneous cofactors including the densities $\rho_s^2/\rho\Delta\rho$ and $\rho_s\Delta\rho/\rho^2$ of the media, these densities having been derived from two independent parameters ρ_s and ρ. The homogeneous cofactors can be presented in the form $(\rho_s/\Delta\rho) \times (\rho_s/\rho)$ and $(\rho_s/\rho)(\Delta\rho/\rho)$ and here $\Delta\rho/\rho$ is identically ρ_s/ρ. As an independent characteristic $\Delta\rho/\rho$ in the general case is dropped from the criteria (for $\rho_s \approx \rho$, $\Delta\rho/\rho_s \approx 0$, for $\rho_s \gg \rho$, $\Delta\rho/\rho_s \approx 1$). Of the remaining criteria, $\Delta\rho/\rho$ is a more general characteristic than ρ_s/ρ, although for the gas–solid system $\Delta\rho/\rho = \rho_s/\rho$ because $\rho_s/\rho \gg 1$.

On the basis of the transformation rules in the theory of similarity [10] we finally obtain the dimensionless variables: the number is the dimensionless rate $(v^3/g\nu)^{1/3} = v/(g\nu)^{1/3}$, the criterion is the dimensionless diameter $(gl^3/\nu^2)^{1/3} = l(g/\nu^2)^{1/3}$, and the parametric criterion $\Delta\rho/\rho$.

In determining the critical fluidization rate we find that the two conditions adopted above make it possible to carry out the substitution $v = w_{cr}$, since $u = 0$ and $l = d_{eq}$. Then, finally, the general form of the dimensionless equation

$$W = f(D_m, \quad \Delta\rho/\rho),$$

$$W \equiv w_{cr}/\sqrt[3]{g\nu} \quad \text{and} \quad D_m \equiv d_{eq}\sqrt[3]{g/\nu^2}.$$

We note that the representation of the number W and of the criterion D_m without exponents for the rate and the diameter fundamentally does not violate the form of the dimensionless complex [10]. At the same time, when calculating D_m in terms of

$$d_{eq} = 1 : \sum_{i=1}^{k}\frac{g_i}{d_i} \quad \text{and} \quad d_i = \sqrt[3]{\frac{2\,\varnothing_1^2\,\varnothing_2^2}{\varnothing_1+\varnothing_2}}$$

we eliminate the artificial exaggeration of the unavoidable errors in calculating the equivalent diameter according to the specified mesh composition of the initial narrow fractions after classification of the mixture on screens with a grid passage dimension $\varnothing_1$ and a grid nonpassage dimension $\varnothing_2$. For particles calibrated as $d_{eq} = d_g$ we find that the error in the determination of the dimension has also been eliminated.

In the case of fluidization by means of a gas under pressure the equation of motion experiences no changes, since the gas pressure acts on the particles in all directions. The effect of both pressure and temperature must be reflected exclusively in the physical constants of the fluidizing agent and they must be included in the complexes W, D_m, and $\Delta\rho/\rho$.

The available data on the critical rates of fluidization for monodisperse beds on passage through the bed of liquids and various gases at normal and high pressures and temperatures [8,13] were processed in the manner proposed above and are shown in the figure. The approximation equation has the form

$$W = cD_m^n\left(\frac{\rho_s - \rho}{\rho}\right)^{0.6},$$

where for $D_m \le 3$, $c = 0.025$, and $n = 1.3$, while for $D_m \ge 3$, $c = 0.045$, and $n = 0.765$.

The maximum deviation in the experimental data amounts to ±15%. We note that the points corresponding to the penetrations of the beds consisting of irregularly shaped particles (naturally alloyed iron ore) at a temperature of up to 400° C lie on the approximation curve.

NOTATION

P is the pressure force on a particle; **G** is the Archimedean force; $\mathbf{F}_f$ is the friction force; $\mathbf{F}_d$ is the drag force; $\mathbf{F}_c$ is the force of collision; I is the inert force; $\mathbf{F}_f$ is the lifting force; ρ_s is the density of the solid material; w is the gas velocity, u is the absolute velocity of the particles; v is the relative velocity of the particles; μ is the coefficient of viscosity of the medium; d_g and d_{ch} are the diameters of grain and channel, respectively; d_{eq} is the equivalent diameter of the particles.

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